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LETTER TO THE EDITOR

The spin freezing transition in the disordered electron problem

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Abstract. We consider a generic model for a disordered Fermi liquid. By means of an infinite resummation of the loop expansion we show that there is a phase transition involving the spin degrees of freedom which can be identified with a new fixed point in the renormalisation group flow equations. The critical exponents obtained for $2 < d < 4$ are apparently exact. The large-disorder phase is described as an incompletely frozen spin phase with no long-range order and subdiffusive spin transport. The results are in good agreement with experiments on Si:P.

Interesting developments have recently occurred in the problem of disordered interacting electrons. The field theoretic formulation of the problem [1] allows for the application of renormalisation group (RG) techniques. The interpretation of the resulting scaling equations in the absence of spin-flip mechanisms has long posed a problem because of their failure to display an obvious fixed point (FP). It was noticed in [2] that they might possess an unconventional FP where the resistance, g, scales to zero, and a triplet interaction amplitude, γ_t, scales to infinity in such a way that their product, $y = g\gamma_t$, is finite at a FP $y^* = 4\varepsilon + O(\varepsilon^2)$ in $d = 2 + \varepsilon$ dimensions. In [2] it was suggested that this FP describes a metal–insulator transition (MIT). The present authors used power counting to prove (within logarithmic accuracy) the existence of this FP to all orders in a loop expansion, and to show that the exponent of the electrical conductivity σ vanishes [3, 4]. However, a RG analysis at two-loop order revealed [4] that the $y^* = 4\varepsilon$ FP is suppressed by logarithmic terms. Trouble with this FP could already be anticipated at one-loop order, where the renormalised two-point vertex functions show a violation of extended dynamical scaling. The suppression of this 'weak-scaling FP' ruined the prospect of finding an answer to the questions about the existence and the nature of the expected instabilities in the transport properties of this model. The established techniques seemed to be exhausted without having led to a solution of the problem.

In this letter we use a different technique, namely a direct infinite resummation of the loop expansion, to derive non-linear integral equations that describe spin transport in the limit $g \to 0$, $y =$ constant. These equations display a phase transition from a spin diffusive phase to one where the spin diffusion coefficient at small frequencies Ω behaves like $D_s(\Omega) \sim \Omega^{(d-2)/2}$ for $2 < d < 4$. There is thus no spin localisation, but spin transport is subdiffusive, and we call this the incompletely frozen (IF) spin phase. We then show that a careful RG analysis yields a second (at one-loop order) FP in the limit $g \to 0$, $\gamma_t \to \infty$, at $y^* = \varepsilon + O(\varepsilon^2)$. This FP shows dynamical scaling. It is immune to the logarithmic problems which suppress the weak-scaling FP [4], while the results of [3], which relied only on the limit $g \to 0$, $y =$ constant, still apply. We analyse the critical behaviour at the new 'dynamical scaling FP', and conclude that it can be identified with the phase transition we find in the integral equations. We thus have a complete and consistent description of a spin instability which leads, with increasing disorder, from spin diffusion to the IF spin phase, while the charge transport remains uncritical. In the following we present our main results and sketch their derivation. A complete account of this work will be published elsewhere [5].

In the limit $g \to 0$, $y =$ constant one can show [5] that perturbation theory [4] simplifies so much that one can obtain an expression for the frequency- and wavenumber-dependent spin diffusivity to *all* orders in the loop expansion:

$$D_s(\boldsymbol{k}, \Omega) = D_s^0 - \mathrm{i}G \int \mathrm{d}p \frac{1}{p^2|\boldsymbol{p}+\boldsymbol{k}|^2} \bigg[\int_0^\infty \mathrm{d}\omega\, \chi(\boldsymbol{p}, \omega)\chi(\boldsymbol{p}+\boldsymbol{k}, \omega+\Omega) - \frac{1}{4}\bigg(\int_0^\Omega \mathrm{d}\omega + \int_\Omega^\infty \mathrm{d}\omega \frac{\Omega}{\omega} \bigg) \chi_c(\boldsymbol{p}, \omega)\chi(\boldsymbol{p}+\boldsymbol{k}, \omega+\Omega) \bigg]. \quad (1a)$$

Here

$$\int \mathrm{d}p = \int \mathrm{d}\boldsymbol{p}/(2\pi)^d$$

and G is the (bare) resistivity, which plays the role of a disorder parameter. D_s^0 is the bare (i.e. Boltzmann) spin diffusion constant. $\chi(\boldsymbol{p}, \omega)$ is a retarded susceptibility

$$\chi(\boldsymbol{p}, \omega) = p^2/(-\mathrm{i}\omega/D(\omega) + p^2). \quad (1b)$$

$D(\omega)$ is the diffusion coefficient associated with the basic diffusion pole in the field theory [6, 4], and it is related to a frequency or temperature renormalisation factor [1]. Its physical meaning is that $D(\omega = 0)$ determines the specific heat coefficient C/T [7]. Finally, χ_c is the charge density susceptibility. It is given by (1*b*) with $D(\omega)$ replaced by the charge diffusion constant D_c. We note that D_c is uncritical [3], and therefore frequency independent and given by its Boltzmann value. Returning to perturbation theory for $D(\omega)$ [4], we can write it to all orders as

$$\frac{1}{D(\Omega)} = \frac{1}{D^0} + \tfrac{3}{8}G \int \mathrm{d}p \frac{1}{\Omega} \int_0^\Omega \mathrm{d}\omega \left(1 - \frac{\omega}{\Omega}\right) \frac{1}{-\mathrm{i}\omega + p^2 D_s(\boldsymbol{p}, \omega)}. \quad (2)$$

The bare diffusion constant D^0, as well as D_c, are related to D_s^0 by means of the Fermi liquid parameters F_0^s, F_0^a, and F_1^s. We add a few remarks: (i) the equations can be obtained from perturbation theory [4] as a particular way of dressing the one-loop result; (ii) it can be shown [5] that (1) is exact in the limit considered [8]; (iii) equation (2) correctly reproduces perturbation theory [4] up to and including at least two-loop order—we will further comment on the validity of (2) after our RG analysis below; (iv) the equations are causal in the sense that any causal choice for $D(\omega)$ and $D_s(\boldsymbol{p}, \omega)$ as input will yield a causal $D_s(\boldsymbol{k}, \Omega)$ and $D(\Omega)$, respectively [9].

To solve (1), (2), we first observe that (1*a*) implies for the spin diffusion constant $D_s = D_s(k = 0, \Omega = 0)$ a structure

$$D_s/D_s^0 = 1 - F(G) \tag{3}$$

where F is a complicated function of G, which in turn depends on D_s itself. F is positive definite, and is expected to be monotonically increasing with G. If $1 - F(G)$ has a zero at $G = G_c$, D_s will vanish at this point. We have performed numerical calculations [5], which show that this is indeed the case with a finite G_c. Let us denote the dimensionless distance from G_c by $t \equiv (G_c - G)/G_c$. Then D_s will vanish like $D_s \sim t^\gamma$ with an exponent

$$\gamma = 1. \tag{4}$$

Since (1) is exact for small D_s, this value for γ should be exact [10]. For $t > 0$, D_s at small k and Ω reads

$$D_s(k, \Omega) = D_s + a_s\Omega^{(d-2)/2} + b_s k^{d-2} \qquad (t > 0) \tag{5}$$

where a_s and b_s are slowly varying functions of G. Spin transport is diffusive with a long-time tail. The critical point, $t = 0$, is characterised by

$$D_s^0 = \mathrm{i}G \int \mathrm{d}p \frac{1}{p^4} \int_0^\infty \mathrm{d}\omega |\chi(\boldsymbol{p}, \omega)|^2 \qquad (t = 0). \tag{6}$$

Expanding (1*a*) and (2) in k and Ω, we find

$$D_s(k, \Omega) = a_s\Omega^{-\eta/z} + b_s k^{-\eta} \qquad (t = 0) \tag{7a}$$

with η and z the usual static and dynamic exponents. Their values are

$$\eta = 2 - d \tag{7b}$$

$$z = 2. \tag{7c}$$

Again we expect these exponents to be exact. This can be corroborated as follows. The divergence of the spin susceptibility $\chi_s \sim 1/D_s$, equation (4), is not sufficiently strong to lead to a spontaneous magnetisation. The exponent of the magnetisation, β, must therefore be zero. Hyperscaling then yields (7*b*). Equation (7*b*) also implies a correlation length exponent

$$\nu = 1/(d - 2). \tag{7d}$$

For $t < 0$, i.e. $G > G_c$, D_s will be zero and (6) must still hold [11]. We are thus led to the conclusion that (7) hold for $t < 0$ as well. This is the IF spin phase. For $D(\Omega)$, we find in the spin diffusive phase

$$D(\Omega) \sim D + a\Omega^{(d-2)/2} \qquad (t > 0) \tag{8a}$$

where $a \sim D_s^{-d/2}$, and the range of validity of (8*a*) shrinks to zero as $t \to 0$. In this limit, D vanishes as

$$D \sim (\ln(1/t))^{-1} \qquad (t \to 0). \tag{8b}$$

If we assign D an exponent κ as in [4], we have $D \sim t^\kappa$ with

$$\kappa = 0. \tag{8c}$$

At the critical point and in the IF spin phase we find

$$D(\Omega) \sim (\ln(1/\Omega))^{-1} \qquad (t \leqslant 0) \tag{9}$$

for small Ω. The specific heat coefficient C/T thus diverges at the critical point and in the IF spin phase, but only logarithmically.

These results provide us with a puzzle, namely how to reconcile them with our previous RG analysis [4]. Equations (4)–(8) suggest the existence of a dynamical scaling FP. However, in [4] we found only the weak-scaling FP with $\gamma = 4$, $\kappa = 3$, which furthermore turned out to be logarithmically suppressed at two-loop order. To clarify this point, let us reconsider the RG. We define coupling constants $\mathcal{H}$ and $\mathcal{K}_t$ such that the Gaussian vertices $\Gamma_0^{(2)}$ and $\Gamma_t^{(2)}$ read [4],

$$\Gamma_0^{(2)} = (1/G)(k^2 + \mathcal{H}) \tag{10a}$$

$$\Gamma_t^{(2)} = (1/G)(k^2 + \mathcal{H} + \mathcal{K}_t). \tag{10b}$$

Here $\mathcal{H} = GH\Omega_M$, $\mathcal{K}_t = GK_t\Omega_M$, where Ω_M is the external Matsubara frequency, and H and K_t are the frequency renormalisation factor and triplet interaction amplitude, respectively, defined in [4]. In terms of H and K_t, we have $D^0 = 1/GH$, and $D_s^0 = 1/G(H + K_t)$. Notice that $\mathcal{H}$ plays the role of the symmetry-breaking external field, and that both $\mathcal{H}$ and $\mathcal{K}_t$ are proportional to Ω_M. We define dimensionless renormalised fields $h = b^2\mathcal{H}^R$, $k_t = b^2\mathcal{K}_t^R$, where b is the RG length scale parameter, and derive flow equations for h and k_t. For our present purpose it is sufficient to go to one-loop order, and the salient point can be seen most clearly if we use a momentum-shell RG. From perturbation theory [6, 4] we obtain

$$\mathrm{d}h/\mathrm{d}\log b = h(2 + \tfrac{3}{4}yf_1(k_t)) \tag{11a}$$

$$\mathrm{d}k_t/\mathrm{d}\log b = k_t(2 + yf_0(h)) \tag{11b}$$

$$\mathrm{d}y/\mathrm{d}\log b = -\varepsilon y + \tfrac{1}{4}y^2(4f_0(h) - 3f_1(k_t)) \tag{11c}$$

with $y = g\,k_t/h = g\gamma_t$. g scales like $b^{-\theta}$, with $\theta = \varepsilon$ exactly [4]. The functions f_0 and f_1 are given by

$$f_0(x) = (1/x)\ln(1 + x) \tag{11d}$$

$$f_1(x) = (2/x)[(1 + 1/x)\ln(1 + x) - 1]. \tag{11e}$$

We expect a phase transition only at zero external field, so we look for a FP where $h^* = 0$, and $f_0(h^*) = 1$. For k_t, we have two possibilities. Since $\Omega_M(b = 1) = 0$ at the phase transition, the most obvious choice is $k_t^* = 0$. This leads to $y^* = 4\varepsilon$, $\gamma = 4 + O(\varepsilon)$, and $\kappa = 3 + O(\varepsilon)$. This is the weak-scaling FP discussed extensively before [4]. However, equation (11b) also allows for a FP at $k_t^* = \infty$. Then $f_1(k_t^*) = 0$, so $y^* = \varepsilon$, $\gamma = 1$, and $\kappa = 0$. This new FP is consistent with the phase transition we found in (1), (2). It can also be analysed by means of the field theoretic RG employed in [4]. One has to keep in mind, however, that conventional minimal subtraction breaks down in the limit $k_t \to \infty$. The limit can be handled by using generalised minimal subtraction or a related scheme [12]. One then obtains results equivalent to (11). By considering two-loop order, it becomes clear that the logarithmic problems that suppressed the weak-scaling FP are absent in the limit $k_t \to \infty$, and that dynamic scaling holds. We conclude that the dynamical scaling FP $(h, k_t, y)^* = (0, \infty, \varepsilon)$ exists to all orders, and can be identified with the phase transition described by (1), (2). In particular, the RG approach is consistent with a scaling equation for D_s,

$$D_s(t, k, \Omega) = b^{-\varepsilon} D_s(tb^{\varepsilon}, kb, \Omega b^2)$$

which in turn is consistent with (4) and (7). Note that the dimension of Ω is two. This is due to the fact that Ω always appear in the combination $g\Omega$ with $g \sim b^{-\varepsilon}$, which reduces the dimension of Ω from d to 2.

We add some remarks concerning the nature of the dynamical scaling FP.

(i) Although a phase transition exists only in the limit $\Omega_M(b = 1) \to 0$, this limit can be approached in different ways. The existence of two FPs (at one-loop order) is a reflection of this ambiguity. If we assume that the RG scale factor b is of the order of the correlation length ξ, we can characterise the two FPs as follows. If the initial or physical frequency $\Omega_M(b = 1)$ goes to zero as ξ^{-x} with $2 < x < d$, then $f_0(\hbar^*) = 1$, and $f_1(\hbar_t^*) = 0$, and the dynamical scaling FP is obtained. If, on the other hand, $x > d + 3\varepsilon$ the weak-scaling FP is found.

(ii) We expect the exponent $\kappa = 0$ to be exact. In [4] we derived an effective theory of the vertex $\Gamma_0^{(2)}$, which was Gaussian plus correction terms which formally vanish near the FP. The only reasons for non-trivial renormalisations of $\hbar$ were diagrammatic 'accidents' which had to do with lack of interchangeability of limits, and which rendered the formal arguments invalid. Inspection shows [5] that none of these accidents arises in the limit $\hbar_t \to \infty$. Near the dynamical scaling FP, the effective theory for $\Gamma_0^{(2)}$ is therefore Gaussian, and $\hbar$ has its bare dimension. This can also be concluded from the scaling law $\kappa = \gamma - \nu\varepsilon$ which follows from y = constant.

(iii) In (8), (9) we found logarithmically critical behaviour of $\hbar$ or D. While this is consistent with $\kappa = 0$, we have to wonder why the logarithm does not appear in the RG. Perturbation theory [5] shows that the logarithmic term is indeed absent order by order in the double expansion in y and $1/(d-2)$, and therefore cannot be found by RG techniques. This is also consistent with the logarithm being independent of dimensionality. It is only the infinite resummation, equation (2), that produces this behaviour. We also note that the existence of the logarithm is tied to the fact that the exponent $\eta = 2 - d$ is exact. It follows that the critical behaviour of D is outside our well controlled perturbative scheme, and we do not know if it is exact. This also holds for our result that the behaviour at small k and Ω in the entire IF spin phase is identical to that at the phase transition. In addition, our 'exact' exponents may be subject to non-perturbative corrections such as the logarithmic correction to κ discussed above.

Let us finally briefly discuss the implications of our results for experiments at finite temperatures. Essentially the discussion given in [4] still applies with two modifications. First, we now have a *bona fide* FP, and no breakdown of scaling is predicted. Second, the exponents $\kappa = 0$ and $\nu = 1/(d-2)$ are now known exactly. At the critical point and in the IF spin phase, we find the specific heat to behave as $C \sim T \ln T$. This provides a very good fit [4] for the deviations from linear behaviour seen in the cleanest sample ($n/n_c = 1.25$) of [13]. The spin susceptibility χ_s diverges like $\chi_s \sim T^{-\mu}$ with

$$\mu = (d-2)/2. \tag{12}$$

Equation (12) is in good agreement with the low-temperature behaviour of the cleanest sample in [14]. With further increasing disorder we expect mode–mode coupling effects to restore the coupling of the charge to the spin degrees of freedom. This will modify the IF spin phase away from the critical point, and eventually lead to a MIT. Close to the MIT and in the insulator our results do not agree with the experiments. This is presumably due to the restored coupling between spin and charge. In [15] a T-dependence of χ_s was found at $n/n_c \simeq 1.4$ that appears to be much stronger that $\sqrt{T}$, and that becomes weaker as the MIT is approached. This is in disagreement with the data of [14]. We do not know the reason for this discrepancy. Further experiments in the metallic phase to look for the dynamical scaling critical point would be very helpful.

In conclusion, we have found a new phase transition in disordered electron systems. The critical exponents have been obtained exactly. The large-disorder phase is described as an incompletely frozen spin phase which is not localised, but displays subdiffusive spin transport. Comparison with experimental data gives good agreement.

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[8] This statement holds for small k and Ω. Outside of this regime there are many corrections that are not in our theory. These corrections are irrelevant for the critical behaviour discussed in this paper.
[9] Strictly speaking, $1/D(\omega)$ is not a causal function. However, the difference between $1/D(\omega)$ and a causal function appears in the propagators only at higher order in p^2 and ω. For the same reason, D_s and D at large frequencies approach their bare values rather than zero. If desired, this can be easily remedied to give D_s and D, which are strictly causal and obey the f sum rule [5]. For our present purposes the point is irrelevant; see [8].
[10] $\gamma \neq 1$ would occur only if $F(G)$ were singular at $G = G_c$, or if the linear term in the Taylor expansion vanished. Generally speaking this is unlikely, and it is not the case with F given by (2).
[11] This is certainly not the case with the equations as written. However, the criterion given by (6) depends *inter alia* on the large-frequency behaviour of $D(\omega)$, over which we have no control. The situation here is very similar to the one in conventional phase transitions, where the transition point depends on all details of the system and cannot be calculated without solving the model exactly, in contrast to the critical behaviour. We note that in our case the complete region $t < 0$ is critical in some sense since there is no long-range order.
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